

AC-induced superfluidity

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We argue that a system of ultracold bosonic atoms in a tilted optical lattice can become superfluid in response to resonant AC forcing. Among others, this allows one to prepare a Bose–Einstein condensate in a state associated with a negative effective mass. Our reasoning is backed by both exact numerical simulations for systems consisting of few particles, and by a theoretical approach based on Floquet–Fock states.

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I. INTRODUCTION

The study of ultracold atoms in optical lattices by now has opened up a promising new area of research on the borderlines between atomic and molecular physics, quantum optics, and condensed-matter physics: Many-body systems of paradigmatic importance to condensed-matter theory are being experimentally realized, with the precision and flexibility offered by quantum optical tools, by means of interacting ultracold atoms subjected to periodic potentials generated by standing waves of laser light. A hallmark example along this line is provided by the successful implementation of the Bose–Hubbard Hamiltonian [1, 2], and the observation of the predicted quantum phase transition from a superfluid to a Mott insulator [3, 4, 5]. Extrapolating these developments, it has been suggested to employ fermionic atoms confined by optical lattices in order to achieve a laboratory realization of the (fermionic) Hubbard model, and thus to tackle questions concerning high- T_c -superconductivity by performing measurements on that system [6], while an exact numerical treatment of the very model lies still beyond existing computational resources.

In this letter, we propose to investigate time-dependent many-body dynamics of ultracold bosonic atoms in a one-dimensional optical lattice subjected to both a static tilt and an additional time-periodic (AC) drive. We argue that such a system exhibits an effect which has no immediate solid-state analogue, namely, the appearance of a superfluid state in response to resonant forcing. We first discuss the underlying mechanism in a general manner, before presenting numerical calculations for small systems, and a quantum Floquet theoretical approach which both support our deductions. Finally we provide parameter estimates concerning the experimental verification of this effect.

II. THE BASIC MECHANISM

Ultracold bosonic atoms in a sufficiently deep one-dimensional optical lattice with M sites are described, to good approximation, by the Bose–Hubbard model [1, 2]

$$\hat{H}_0 = -J \sum_{\ell=1}^{M-1} \left(\hat{b}_{\ell}^{\dagger} \hat{b}_{\ell+1} + \hat{b}_{\ell+1}^{\dagger} \hat{b}_{\ell} \right) + \frac{U}{2} \sum_{\ell=1}^M \hat{n}_{\ell} (\hat{n}_{\ell} - 1), \quad (1)$$

where \hat{b}_{ℓ}^{\dagger} (\hat{b}_{ℓ}) denotes the creation (annihilation) operator for an atom in the Wannier state located at the ℓ th lattice site, $\hat{n}_{\ell} = \hat{b}_{\ell}^{\dagger} \hat{b}_{\ell}$ is the number operator for that site, the hopping matrix element $J > 0$ parametrises the strength of tunneling between adjacent sites, and $U > 0$ quantifies the repulsion energy of a pair of atoms occupying the same site. For vanishing interaction strength, that is, for $U/J \rightarrow 0$, all N atoms condense into the lowest Bloch state of the lattice for temperature $T \rightarrow 0$, giving rise to a superfluid ground state $|\text{SF}\rangle = \frac{1}{\sqrt{N!}} \left(\frac{1}{\sqrt{M}} \sum_{\ell=1}^M \hat{b}_{\ell}^{\dagger} \right)^N |0\rangle$, where $|0\rangle$ denotes the vacuum state. On the other hand, for vanishing tunneling contact, $U/J \rightarrow \infty$, the ground state is the Mott insulating state $|\text{MI}\rangle = \prod_{\ell=1}^M \frac{(\hat{b}_{\ell}^{\dagger})^n}{\sqrt{n!}} |0\rangle$, assuming integer filling $n = N/M$. In the limit of a chain consisting of infinitely many sites, $M \rightarrow \infty$, a sharp transition between the superfluid regime with (quasi) long-range order and the Mott-insulating regime occurs at a critical value $(U/J)_c$, accompanied by the emergence of a finite gap between the energy of the ground state and those of the excited states [1]. For a filling factor of one particle per site, $N/M = 1$, one finds $(U/J)_c \approx 3.4$ [7]. Remarkably, even calculations performed for quite small systems show a precursor of this quantum phase transition: In Fig. 1 we display the energy eigenvalues for a system with $N = M = 5$, as functions of the scaled interaction strength U/J . There is, of course, no sharp transition in such a small system, but the gradual splitting-off of the ground-state energy is clearly discernible.

When the Bose–Hubbard system (1) with $M \rightarrow \infty$ is subjected to both a static tilt amounting to an energy difference per site of K_0 , and to an AC force of frequency ω and amplitude K_{ω} , the total Hamiltonian

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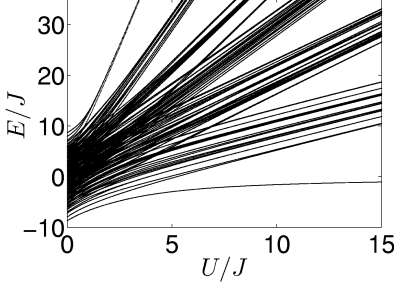


FIG. 1: Energy spectrum versus interaction strength U/J for a small, untilted and undriven Bose-Hubbard-system (1) with $N = 5$ particles on $M = 5$ sites. The splitting-off of the ground state around $U/J \approx 4$ is a precursor of the quantum phase transition that occurs in an infinitely large system.

becomes $\hat{H}(t) = \hat{H}_0 + \hat{H}_1(t)$, where

$$\hat{H}_1(t) = (K_0 + K_\omega \cos(\omega t)) \sum_{\ell=1}^M \ell \hat{n}_\ell. \quad (2)$$

Assuming first $K_\omega = 0$, the static force effectuates a splitting of the single-particle Bloch band into a sequence of single-particle states with equidistant energies $E_\ell = K_0 \ell$ forming the so-called Wannier-Stark ladder [8]. The corresponding single-particle orbitals are associated with creation operators $\hat{c}_\ell^\dagger = \sum_i J_{i-\ell}(2J/K_0) \hat{b}_i^\dagger$, where $J_\alpha(z)$ is a Bessel function of order α . Since $|J_0(2J/K_0)|^2 = 1 - 2(J/K_0)^2 + \mathcal{O}[(J/K_0)^4]$ for $J/K_0 \ll 1$, one has $\hat{c}_\ell^\dagger \approx \hat{b}_\ell^\dagger$ when $K_0 \gg J$, so that the Wannier-Stark states coincide approximately with the original Wannier states, indicating that a strong tilt effectively destroys the tunneling contact between adjacent lattice sites.

The principle we are exploiting in the present proposal relies on the fact that a tunneling contact disabled by a uniform tilt can be partially restored when the system is driven resonantly, *i.e.*, when the driving frequency ω in eq. (2) is chosen such that

$$K_0 \approx \nu \hbar \omega \quad (3)$$

with integer ν , so that the energy of ν quanta $\hbar \omega$ bridges the energy difference between adjacent rungs of the Wannier-Stark ladder [9]. In that case the *driven* Bose-Hubbard system behaves approximately, in a time-averaged sense, like an *undriven* system with a modified hopping matrix element

$$J_{\text{eff}} = (-1)^\nu J_\nu(K_\omega/\hbar \omega) J, \quad (4)$$

provided the frequency is sufficiently high, so that both $\hbar \omega \gg U$ und $\hbar \omega \gg J$. While such a modification of the tunneling contact is reminiscent of the AC-Josephson-effect [10, 11], here the many-body interaction has a remarkable consequence: When $|U/J_{\text{eff}}|$ becomes smaller than $(U/J)_c$, the system becomes superfluid despite the presence of the strong tilt. Observe that this scenario

also includes pure AC-forcing with $K_0 = 0$ as a special case for $\nu = 0$. For this case, which has been considered theoretically before [12, 13], the expected J_0 -modification of the hopping matrix element has been beautifully confirmed in a recent experiment [14], thus demonstrating the feasibility of exposing a Bose-Einstein condensate in an optical lattice to strong AC forcing without destroying its phase coherence.

In order to verify the existence of AC-induced superfluidity, we propose the following experimental protocol: (i) Initially the ultracold atoms are prepared in the superfluid ground state of a shallow one-dimensional optical lattice. Then the interaction parameter U/J is ramped up by increasing the depth of the lattice. At the critical parameter $(U/J)_c$ the system enters the Mott-insulator regime, and the particles localise at the lattice sites. For large values of U/J , the system approximately reaches the extreme Mott state |MI>, again assuming integer filling $n = N/M$. (ii) Next, a strong static tilt is applied, e.g. by accelerating the lattice. Then U/J is switched down again to a small value, such that the system would fall into the superfluid regime if there were no tilt. However, it actually remains in a state close to |MI>, which, in its turn, still remains an approximate stationary state of the Hamiltonian, due to the strong localisation of the Wannier-Stark states. (iii) Finally the resonant AC force is turned on. In order to guide the system adiabatically into the AC-induced superfluid state corresponding to the ground state of the effective undriven Bose-Hubbard Hamiltonian with the modified hopping element (4), that force should be turned on smoothly, starting from $K_\omega/\hbar \omega = 0$.

III. EXACT SIMULATION OF SMALL SYSTEMS

We have computed the exact time evolution of a small system of $N = 9$ particles on $M = 9$ lattice sites, yielding the filling factor $N/M = 1$. To analyze the time-dependent wave function $|\psi(t)\rangle$, we calculate the single-particle quasimomentum distribution

$$\rho(p, t) = \frac{1}{M} \sum_{\ell, j} \exp \left[i \frac{(\ell - j)p}{\hbar/a} \right] \langle \psi(t) | \hat{b}_\ell^\dagger \hat{b}_j | \psi(t) \rangle, \quad (5)$$

where a is the lattice constant, and record it versus $pa/(2\pi\hbar)$ at times t that are integer multiples of the driving period $T = 2\pi/\omega$. This distribution is $2\pi\hbar/a$ -periodic with respect to p ; together with an envelope provided by the momentum distribution of the corresponding Wannier state it yields the momentum distribution of the system that can be measured experimentally by time-of-flight absorption imaging [3, 4, 5]. It allows one to clearly distinguish between the superfluid and the Mott state: Whereas the momentum distribution of a Mott state is rather structureless, a superfluid state with (quasi) long-range phase coherence is characterised by sharp peaks

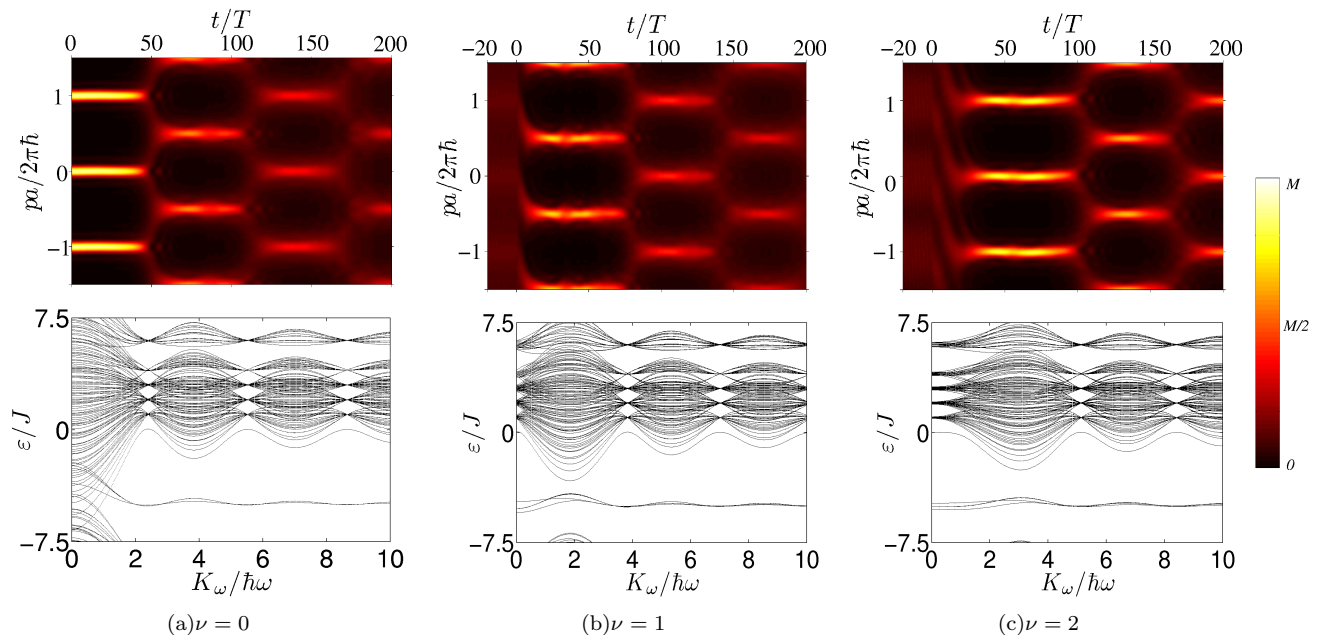


FIG. 2: *Upper row:* Three Brillouin zones of the quasimomentum distribution $\rho(p, t)$, recorded at integer t/T , while the driving amplitude $K_\omega/\hbar\omega$ increases linearly from 0 to 10 between $t = 0T$ and $t = 200T$. Parameters for $t \geq 0T$ are $U/J = 1$ and $\hbar\omega/J = 15$; $N = M = 9$. For $\nu = 0$, the system starts in a superfluid state, corresponding to a sharply peaked distribution. When $K_\omega/\hbar\omega$ passes a zero of J_0 , the maxima of the distribution switch from the centers to the edges of the Brillouin zones, or vice versa. For $\nu = 1$ and $\nu = 2$, the system initially is prepared in a Mott-like state. Nonetheless, the sharp peaks emerging at later times, when $|J_{\text{eff}}|$ is sufficiently large, signal the appearance of AC-induced superfluidity. Again, the peaks switch between the zone centers and the edges at the zeros of the Bessel function J_1 or J_2 , respectively. — *Lower row:* Exact instantaneous quasienergies for the same situations, computed with $N = M = 5$. Observe that the changes in the momentum distributions above are precisely reflected by the band collapses.

at the minima of the effective single-particle dispersion relation $E_{\text{eff}}(p) = -2J_{\text{eff}} \cos(pa/\hbar)$.

Fixing the time scale such that the AC force with high frequency $\hbar\omega/J = 15$ is turned on at $t = 0$, the system is initialised in the ground state of an undriven, untilted lattice with large interaction parameter $U/J = 50$ at time $t = -20T$. Simultaneously the lattice is tilted abruptly by $K_0 = \nu\hbar\omega$, with $\nu = 1$ or 2 . At $t = -10T$ the interaction strength is decreased abruptly to $U/J = 1$. These preparatory steps lead to no significant change of the initial Mott state. That state coincides to good accuracy (overlap ≈ 0.95 or larger) with the ground state $|\text{MI}\rangle$ of the effective system for $K_\omega/\hbar\omega = 0$, i.e., for $J_{\text{eff}} = 0$. For comparison, we also show results obtained for an untilted lattice ($\nu = 0$). In that case, the system simply is initialised at $t = 0$ in its superfluid ground state for $U/J = 1$.

During the time interval from $0T$ to $200T$ the dimensionless driving amplitude $K_\omega/\hbar\omega \equiv z$ is ramped up linearly from 0 to 10. The upper row in Fig. 2 shows the resulting quasimomentum distributions (5), as obtained from numerical solutions of the time-dependent many-body Schrödinger equation. For $\nu = 0$, the system starts in a superfluid state. Accordingly, $\rho(p, t)$ displayed in Fig. 2(a) shows a high contrast right from the outset at $t = 0$, with sharp maxima at the Brillouin zone center,

$pa/(2\pi\hbar) = 0, \pm 1, \pm 2, \dots$. However, when the gradually rising amplitude $K_\omega(t)/\hbar\omega$ passes the first zero of $J_0(z)$ at $z \approx 2.4$, there is a short interval during which $|J_{\text{eff}}|$ is so small, or $|U/J_{\text{eff}}|$ so large, that the system becomes Mott-like [12]. When the superfluid-like state reemerges at later t , the maxima of $\rho(p, t)$ have shifted to the edges of the Brillouin zone. This finding is fully in accordance with the fact that J_{eff} is *negative* between the first and second zero of J_0 , so that the minima of $E_{\text{eff}}(p)$ occur at the zone edges. When the driving amplitude rises even beyond the second zero of $J_0(z)$ at $z \approx 5.5$, the original conditions are restored. We remark that this change of sign of the effective hopping matrix element, which has been clearly observed in the experiment [14], is a nontrivial consequence of the “dressing” of the system achieved through the time-periodic forcing: In effect, this amounts to placing the condensate in a state with a negative effective mass, the sign of that mass being controlled by the forcing strength.

When $\nu = 1$, the system initially is Mott-like at $t = 0T$. However, as soon as $K_\omega(t)/\hbar\omega$ becomes sufficiently large, the tunneling contact is restored, and the system becomes superfluid-like, as witnessed by the sharp feature visible in Fig. 2(b). This is precisely the effect we are focussing on. Observe that here J_{eff} is negative between $z = 0$ and the first zero of $J_1(z)$ at $z \approx 3.8$, so

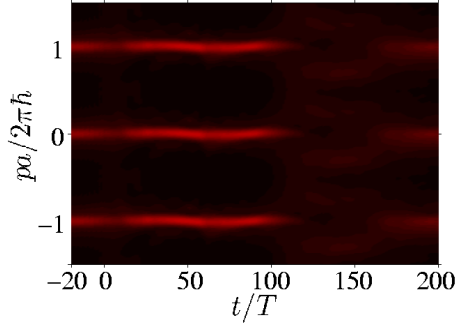


FIG. 3: Momentum distribution for parameters and protocol as in Fig. 2(c), but for an initial state with half filling, $N = 6$ particles on $M = 12$ sites. Shortly after $t = 100 T$, when J_{eff} changes its sign, the signature of superfluidity is lost, since the system is unable to follow the rapidly changing ground state.

that the negative-mass picture now holds even for small driving amplitudes. The results for $\nu = 2$ in Fig. 2(c) likewise follow the expected pattern. Thus, the computations summarised in the upper row of Fig. 2 can be explained in terms of an effective time-independent Bose–Hubbard Hamiltonian (1) equipped with the modified hopping matrix element (4) at any instantaneous value of the driving amplitude, in combination with approximate adiabatic following of the system’s wave function in response to the actual slow rise of that amplitude. This intuitive picture has a striking implication: A Bose–Hubbard system at half filling remains superfluid even for $J/U \approx 0$ [1]. But when the sign of J_{eff} changes, the superfluid ground state changes abruptly, rendering adiabatic following impossible. This is illustrated in Fig. 3, where we have plotted the momentum distribution for the same parameters and protocol as previously considered in Fig. 2(c), but for $N = 6$ and $M = 12$: For times slightly larger than $100 T$, where J_{eff} becomes negative, superfluidity is lost. This expected outcome signals that the high degree of adiabaticity achieved in Fig. 2 for unit filling actually is a nontrivial many-body effect; it hinges on the presence of a Mott-like state when J_{eff}/U is close to zero.

IV. QUANTUM FLOQUET THEORETICAL APPROACH

We now give a formal explanation of the approximate AC-induced modification of the hopping matrix element (4). We separate the full Hamiltonian $\hat{H}_0 + \hat{H}_1(t)$ into a part which is site-diagonal,

$$\hat{H}_{\text{int}}(t) = \frac{U}{2} \sum_{\ell=1}^M \hat{n}_{\ell} (\hat{n}_{\ell} - 1) + \hat{H}_1(t), \quad (6)$$

and a part which describes tunneling between neighbouring sites,

$$\hat{H}_{\text{tun}} = -J \sum_{\ell=1}^{M-1} \left(\hat{b}_{\ell}^{\dagger} \hat{b}_{\ell+1} + \hat{b}_{\ell+1}^{\dagger} \hat{b}_{\ell} \right). \quad (7)$$

We do not require that the resonance condition (3) is met exactly, which would be close to impossible in a laboratory experiment, but rather admit a slight detuning, and choose the integer ν such that $|K_0 - \nu \hbar \omega|$ is minimised. Then one has

$$K_0 = \nu \hbar \omega + \Delta K_0 \quad (8)$$

with $|\Delta K_0| \ll \hbar \omega$. Now the set of Fock states

$$|\{n_{\ell}\}\rangle = \sum_{\ell} \frac{(\hat{b}_{\ell}^{\dagger})^{n_{\ell}}}{\sqrt{n_{\ell}!}} |0\rangle, \quad (9)$$

where $\{n_{\ell}\}$ runs over all admissible sets of occupation numbers of the lattice sites, forms a basis for the diagonalisation of the time-independent Bose–Hubbard Hamiltonian (1). However, when the forcing (2) is turned on with constant amplitude K_{ω} , the full Hamiltonian $\hat{H}(t) = \hat{H}_{\text{int}}(t) + \hat{H}_{\text{tun}}$ is T -periodic in time, with $T = 2\pi/\omega$. In that case the system’s Floquet states, *i.e.*, the complete set of explicitly time-dependent, T -periodic solutions $|u(t)\rangle$ to the eigenvalue equation

$$\left[\hat{H}(t) - i\hbar \partial_t \right] |u(t)\rangle = \varepsilon |u(t)\rangle, \quad (10)$$

take over the role of the stationary states; the wave functions $|\psi(t)\rangle = |u(t)\rangle \exp(-i\varepsilon t/\hbar)$ solve the time-dependent Schrödinger equation. The spectrum of their quasienergies ε is obtained by diagonalising the operator $\hat{H}(t) - i\hbar \partial_t$ in an extended Hilbert space of T -periodic functions, endowed with the scalar product

$$\langle\langle \cdot | \cdot \rangle\rangle \equiv \frac{1}{T} \int_0^T dt \langle \cdot | \cdot \rangle \quad (11)$$

which combines the usual scalar product $\langle \cdot | \cdot \rangle$ with time-averaging [15]. Accordingly, we employ a set of Floquet–Fock states

$$|\{n_{\ell}\}, m\rangle \equiv |\{n_{\ell}\}\rangle \times \exp \left\{ -i \left[\frac{K_{\omega}}{\hbar \omega} \sin(\omega t) + \nu \omega t \right] \sum_{\ell} \ell n_{\ell} + i m \omega t \right\} \quad (12)$$

which diagonalise the part $\hat{H}_{\text{int}}(t) - i\hbar \partial_t$ of the quasienergy operator: By construction, one has

$$\begin{aligned} \langle\langle \{n_{\ell}\}, m | \hat{H}_{\text{int}}(t) - i\hbar \partial_t | \{n_{\ell}\}, m \rangle\rangle \\ = \frac{U}{2} \sum_{\ell} n_{\ell} (n_{\ell} - 1) + \Delta K_0 \sum_{\ell} \ell n_{\ell} + m \hbar \omega. \end{aligned} \quad (13)$$

Observe that the quasienergy spectrum is periodic in ε with period $\hbar\omega$; the integer $m = 0, \pm 1, \pm 2, \dots$ introduced as a Fourier index in the basis states (12) serves to distinguish different Brillouin zones of that spectrum. Employing the identity $e^{iz \sin \varphi} = \sum_{k=-\infty}^{+\infty} e^{ik\varphi} J_k(z)$, one easily obtains

$$\begin{aligned} & \langle \{n'_\ell\}, m' | \hat{b}_\ell^\dagger \hat{b}_{\ell+1} | \{n_\ell\}, m \rangle \\ &= \langle \{n'_\ell\} | \hat{b}_\ell^\dagger \hat{b}_{\ell+1} | \{n_\ell\} \rangle J_{s(m-m')-\nu}(K_\omega/\hbar\omega) \end{aligned} \quad (14)$$

with $s \equiv \sum_\ell \ell(n_\ell - n'_\ell) = +1$, since $\hat{b}_\ell^\dagger \hat{b}_{\ell+1}$ transfers one particle by one site to the left. Note that when taking the adjoint matrix element, both m, m' and $\{n_\ell\}, \{n'_\ell\}$ interchange their roles, so that the sign s changes to -1 and the numerical value of the element remains unchanged. Hence, with respect to the index m the matrix of the quasienergy operator has a simple block structure:

$$\begin{aligned} & \langle \{n'_\ell\}, m' | \hat{H}(t) - i\hbar\partial_t | \{n_\ell\}, m \rangle \\ &= \delta_{m',m} \langle \{n'_\ell\} | \hat{H}_{\text{eff}} + m\hbar\omega | \{n_\ell\} \rangle \\ &+ (1 - \delta_{m',m}) \langle \{n'_\ell\} | \hat{V} | \{n_\ell\} \rangle. \end{aligned} \quad (15)$$

The diagonal blocks ($m = m'$) are just the matrices of a Bose-Hubbard system in the standard Fock basis (9), without AC forcing, but with modified hopping strength (4) and possibly a slight residual tilt, as described by the effective Hamiltonian

$$\begin{aligned} \hat{H}_{\text{eff}} \equiv & -J_{\text{eff}} \sum_{\ell=1}^{M-1} \left(\hat{b}_\ell^\dagger \hat{b}_{\ell+1} + \hat{b}_{\ell+1}^\dagger \hat{b}_\ell \right) + \frac{U}{2} \sum_{\ell=1}^M \hat{n}_\ell (\hat{n}_\ell - 1) \\ & + \Delta K_0 \sum_{\ell} \ell \hat{n}_\ell. \end{aligned} \quad (16)$$

These blocks are shifted against each other by multiples of the “photon” energy $\hbar\omega$, as corresponding to the Brillouin-zone structure of the quasienergy spectrum. They are coupled by off-diagonal blocks provided by an operator \hat{V} which differs from the tunneling term (7) such that the hopping element J is multiplied by a Bessel function $J_{m-m'-\nu}(K_\omega/\hbar\omega)$ for each particle transfer directed to the left, as described by a combination $\hat{b}_\ell^\dagger \hat{b}_{\ell+1}$, and by $J_{m'-m-\nu}(K_\omega/\hbar\omega)$ for each transfer directed to the right, corresponding to $\hat{b}_{\ell+1}^\dagger \hat{b}_\ell$.

The viewpoint advocated before, namely, the description of the forced system $\hat{H}_0 + \hat{H}_1(t) = \hat{H}_{\text{int}}(t) + \hat{H}_{\text{tun}}$ in terms of the effective time-independent Hamiltonian (16), now amounts to neglecting all off-diagonal coupling blocks with $m' \neq m$. This is a viable approximation in the high-frequency regime, where (besides $|\Delta K_0|$) both relevant energy scales J and U are small compared to $\hbar\omega$. Intuitively speaking, this condition guarantees that the AC drive is off-resonant with respect to low-order transitions. As expressed by the scalar product (11), the replacement of the original Hamiltonian by \hat{H}_{eff} essentially relies on the averaging principle.

To substantiate our reasoning, the lower row of Fig. 2 shows exact quasienergy spectra, obtained from numerical solutions of the eigenvalue equation (10) with $N =$

$M = 5$, for the same cases as considered in the dynamical simulations displayed in the upper row of that figure. These spectra are well matched to the effective Hamiltonian (16) with $\Delta K_0 = 0$; in particular, the observed band collapses are accurately located at the zeros of the respective Bessel function J_ν . In the vicinity of these points the quasienergy of the effective ground state separates markedly from the bands of excited states, thus indicating the presence of a Mott-like regime. The width of the quasienergy bands reflects the strength of the effective tunneling contact: For $\nu = 0$, there is no tilt and the contact provided by \hat{H}_{tun} is active for small $K_\omega/\hbar\omega$, resulting in wide bands. In contrast, for $\nu = 1$ and $\nu = 2$ tunneling is hindered by the static tilt, so that superfluidity is enabled by a process akin to photon-assisted tunneling only when the AC drive is sufficiently strong.

Moreover, \hat{H}_{eff} implies a simple scaling property: Quasienergy spectra obtained for different ν and different $K_\omega/\hbar\omega$, but corresponding to the same value of $|J_{\text{eff}}|$, should be almost identical. This fact is confirmed in Fig. 4, where we have plotted spectra for $\nu = 0, 1$, and 2 , and driving amplitudes $K_\omega/\hbar\omega = 1.811, 0.711$ and 1.915 , respectively, chosen such that $|J_{\text{eff}}| = J/3$ in each case. Indeed, the three spectra look strikingly alike; as expected; their gross features are obtained from the energy spectrum shown in Fig. 1 by rescaling the axes of that figure by a factor of $1/3$, and then taking the eigenvalues modulo $\hbar\omega$ to account for the quasienergy Brillouin zones.

Besides the existence of many-body Floquet states, a second key ingredient to our proposal is adiabatic following. In fact, Floquet states respect an adiabatic principle [16]; the initial energy eigenstate evolves into the “connected” Floquet state when the AC drive is turned on sufficiently slowly. However, in the present scenario adiabatic following is endangered by the couplings associated with \hat{V} , which we have neglected when retaining only the diagonal blocks of the quasienergy matrix (15). These couplings lead to many weak resonances among the instantaneous quasienergy levels. Hence, what appears as effectively adiabatic motion actually should be viewed as highly intricate Landau-Zener dynamics at multiple avoided level crossings, so that one may even deteriorate the quality of adiabatic following by reducing the rate of the parameter change in some cases. In the examples shown in Fig. 2, the squared overlap $|\langle \psi_{\text{eff}} | \psi(t) \rangle|^2$ of the true wave function $|\psi(t)\rangle$ with the ground state $|\psi_{\text{eff}}\rangle$ of the instantaneous \hat{H}_{eff} (with $\Delta K_0 = 0$) at integer t/T always remains larger than $0.26 \approx 0.89^N$, $0.64 \approx 0.95^N$, and $0.30 \approx 0.87^N$ for $\nu = 0, 1$, and 2 , respectively, and $N = 9$. The extent to which this adiabatic principle can be exploited to guide the evolution of ultracold atoms in AC-driven optical lattices into certain desired target states is a subject requiring further studies, both experimental and theoretical.

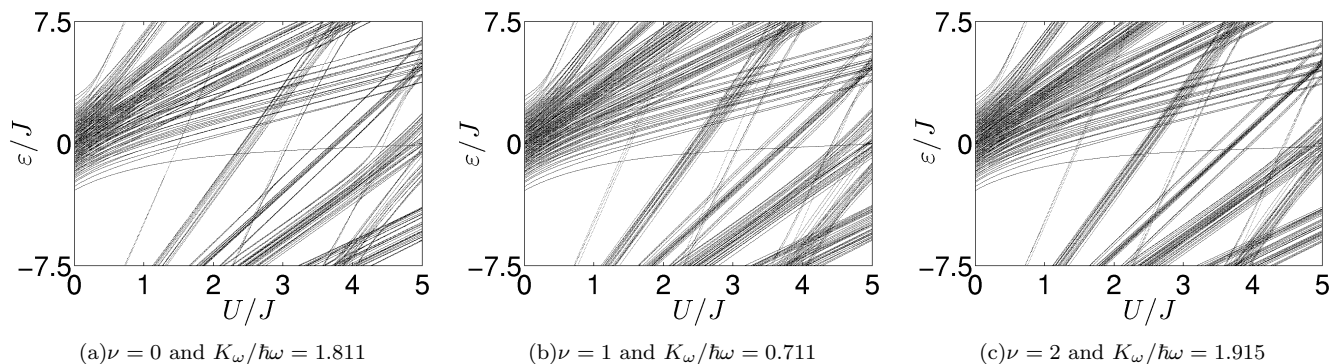


FIG. 4: Exact quasienergy spectra versus interaction strength U/J for a small system ($N = M = 5$) for $\hbar\omega/J = 15$, and $K_0 = \nu\hbar\omega$, with $\nu = 0, 1, 2$. Amplitudes $K_\omega/\hbar\omega$ are chosen such that $|J_{\text{eff}}| = J/3$. Observe that these spectra closely resemble the energy eigenvalues in Fig. 1, when the axes in that figure are rescaled by $1/3$ and the eigenvalues are taken modulo $\hbar\omega$.

V. EXPERIMENTAL FEASIBILITY AND CONCLUSION

In a laboratory experiment with optical lattices, some restrictions must be met in order not to leave the scope of the single-band Bose–Hubbard model: The static tilt must remain weak enough to prevent Zener tunneling on the time scale of the experiment, and both the frequency and the amplitude of the AC force must remain sufficiently low in order not to excite transitions to higher bands. Denoting the energy gap between the lowest two Bloch bands by E_g , this translates into the requirements $U, J \ll K_0, \hbar\omega, K_\omega \ll E_g$, while $U/J < (U/J)_c$. Considering a one-dimensional lattice with a depth of four atomic recoil energies and strong transversal confinement, as in the Zürich experiment [4], one finds $E_g \approx 23 J$ and $U/J \approx 614 a/d$, where a denotes the s -wave scattering length of the atomic species employed, and d is

the lattice constant. Thus, favourable conditions with $U/J \lesssim 1$ are obtained when $a/d \lesssim 0.0016$. This demand appears severe, but not impossible to satisfy.

In closing, we remark that the modification (4) of the hopping matrix element is very similar to the modification of atomic g -factors by oscillating magnetic fields [17], both mathematically and conceptually [18]. Understanding that modification has been crucial for developing the “dressed atom”-picture of atomic physics. Hence, the pioneering experiment [14], having confirmed eq. (4) for $\nu = 0$ with a Bose–Einstein condensate, eventually might lead to an analogous picture of “dressed matter waves”.

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